A Tensor Completion Approach for Fusion of Imaging Data

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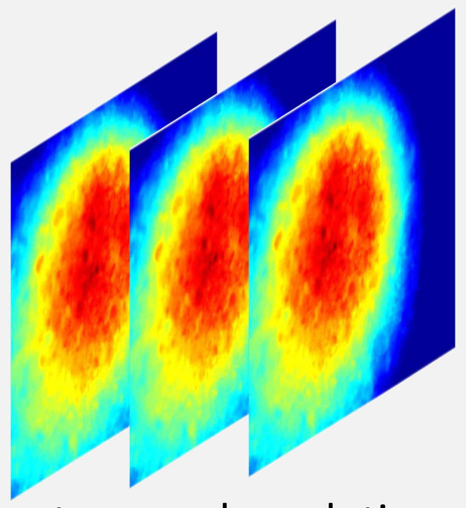
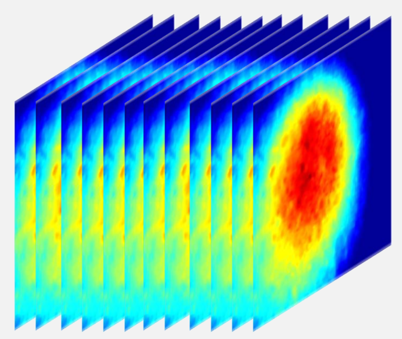
Research Motivation

Fuse different spatial temporal resolution images to obtain high spatial, temporal resolution images



Low temporal resolution, high spatial resolution

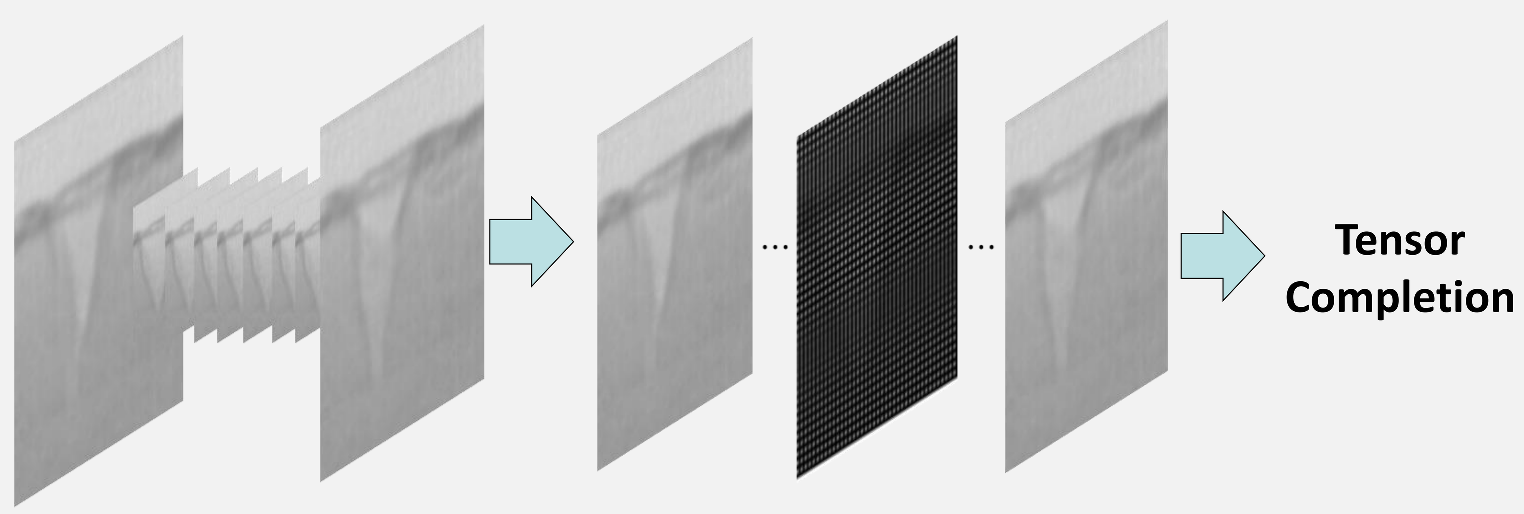


High temporal resolution, low spatial resolution

Research Questions

- How to make use of the information from time sequence?
- How to enhance the edge in the reconstruction image?

Research Framework



Framework on X-ray images with different spatial temporal resolution

Proposed Formulation

Tensor Completion (Liu *et al.*, 2012)

Estimate the values of missing elements of array data

Smooth Sparse PARAFAC tensor completion

Reconstruction Error — Smooth Features — Sparse Features

$$\min_{g,\mathbf{U},\tau,\mathbf{V}} \frac{1}{2}\|\mathcal{T}_\Omega-\mathcal{Z}_\Omega\|_F^2+\sum_{r=1}^{R}\frac{g_r^2}{2}\sum_{n=1}^{N}\rho^{(n)}\left\|\mathbf{L}^{(n)}\mathbf{u}_r^{(n)}\right\|_p^p+\sum_{m=1}^{M}\frac{\tau_m^2}{2}\sum_{n=1}^{N}\phi^{(n)}\left\|\mathbf{v}_m^{(n)}\right\|_1,$$

$$\text{s.t. } \mathcal{Z}=\sum_{r=1}^{R} g_r\mathbf{u}_r^{(1)}\circ\mathbf{u}_r^{(2)}\circ\cdots\circ\mathbf{u}_r^{(N)}+\sum_{m=1}^{M}\tau_m\mathbf{v}_m^{(1)}\circ\mathbf{v}_m^{(2)}\circ\cdots\circ\mathbf{v}_m^{(N)},$$

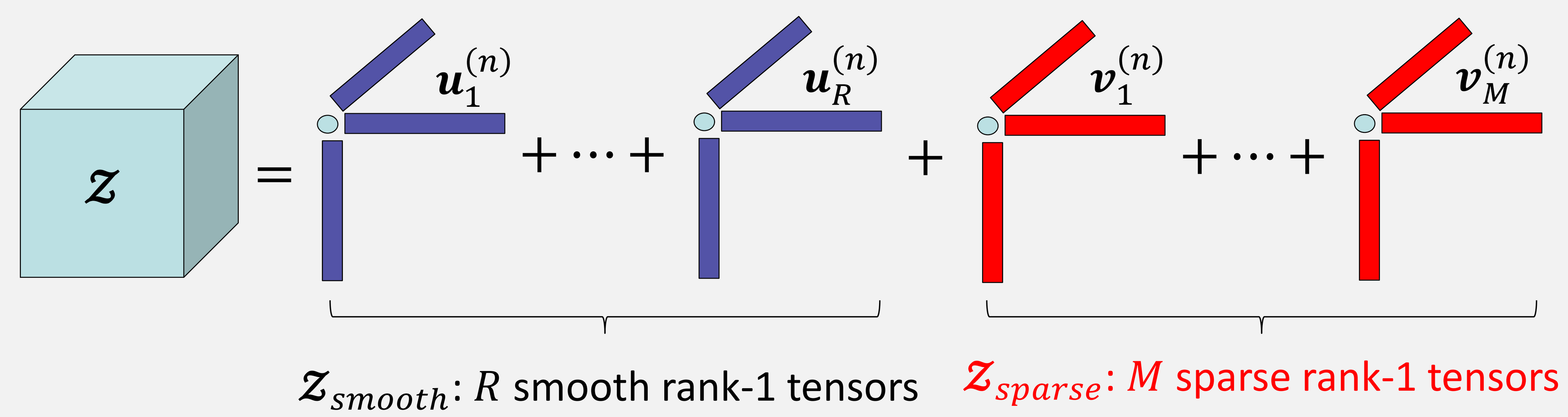
$$\left\|\mathbf{u}_r^{(n)}\right\|_2=1,\left\|\mathbf{v}_m^{(n)}\right\|_2=1,\forall r,n,m$$

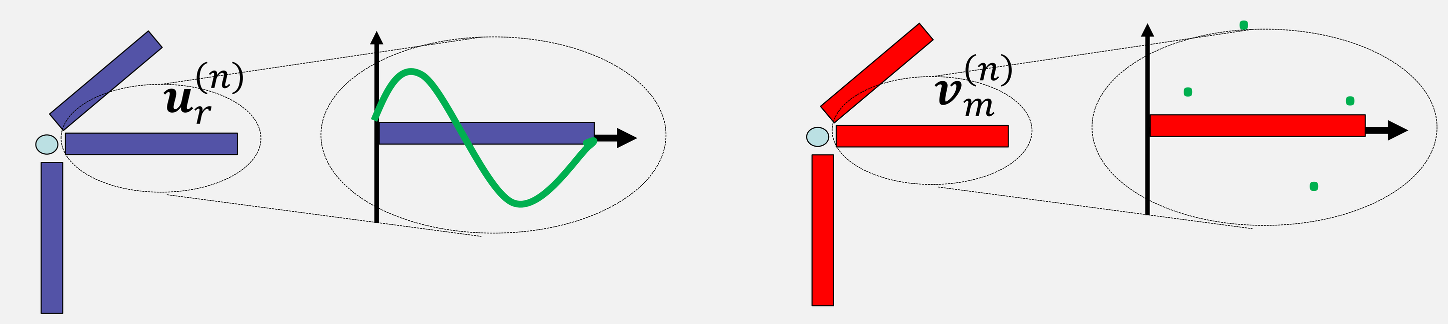
Optimization: Fixed rank Smooth Sparse PARAFAC tensor completion

FR-SSPC is an alternating minimization algorithm

Theorem 1: The FR-SSPC algorithm is a monotonically non-increasing until the objective value converges

More on Formulation





Smoothness in vector space — Sparsity in vector space

Total variation

TV ($p=1$): $\sum_i\left|\mathbf{u}_r^{(n)}(i+1)-\mathbf{u}_r^{(n)}(i)\right|$

Quadratic variation

QV($p=2$): $\sum_i\left|\mathbf{u}_r^{(n)}(i+1)-\mathbf{u}_r^{(n)}(i)\right|^2$

Theorem 2: The regularization terms in the formulation enforce smoothness in $\mathcal{Z}_{smooth}$ and sparsity in $\mathcal{Z}_{sparse}$, specifically,

$$\|\mathcal{Z}_{smooth}\|_{TV}\le\sqrt{\prod_n I_n}\sum_{r=1}^{R}\frac{|g_r|}{2}\sum_{n=1}^{N}\left\|\mathbf{L}^{(n)}\mathbf{u}_r^{(n)}\right\|_1,$$

$$\|\mathcal{Z}_{smooth}\|_{QV}\le R\sum_{r=1}^{R}\frac{g_r^2}{2}\sum_{n=1}^{N}\left\|\mathbf{L}^{(n)}\mathbf{u}_r^{(n)}\right\|_2^2,$$

$$\|\mathcal{Z}_{sparse}\|_1\le\frac{1}{N}\sqrt{\prod_n I_n}\sum_{m=1}^{M}\frac{|\tau_m|}{2}\sum_{n=1}^{N}\left\|\mathbf{v}_m^{(n)}\right\|_1$$

Optimization on R, M

$$\min_{\mathcal{Z}\in\mathbb{SS}(R,M)} R+M,\ \text{s.t.}\ \|\mathcal{T}_\Omega-\mathcal{Z}_\Omega\|_F^2\le\varepsilon.$$

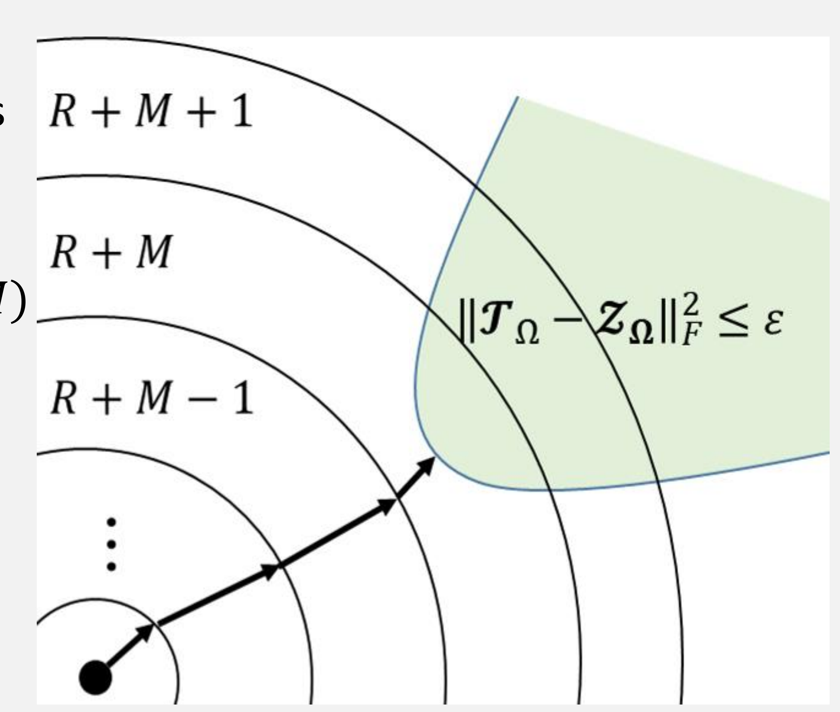
$\mathbb{SS}(R,M)$: set of decompositions which contain R smooth and M sparse components

Define $E(R+M)=\min_{\substack{\mathcal{Z}\in\mathbb{SS}(a,b)\\ a+b=R+M}}\|\mathcal{T}_\Omega-\mathcal{Z}_\Omega\|_F^2$

$E(1)\ge\cdots\ge E(R+M-1)\ge\varepsilon\ge E(R+M)$

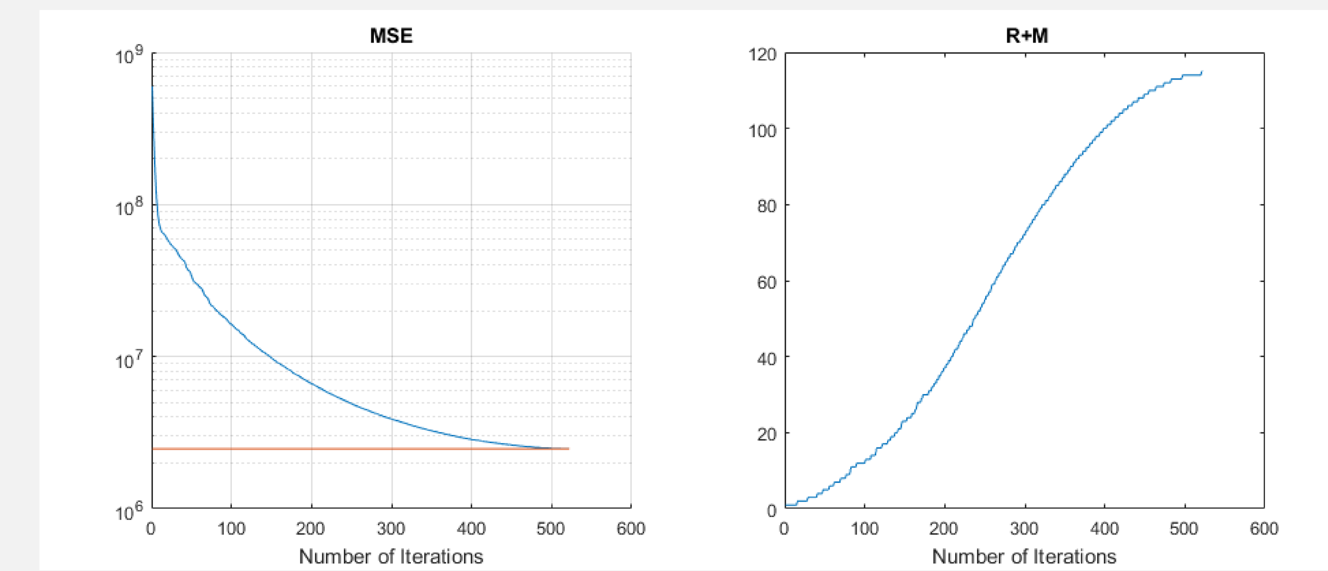
SSPC Algorithm

1: **input:** $\mathcal{T},\Omega,p,\rho,\phi,\varepsilon$
2: $R\leftarrow 0, S\leftarrow 0$
3: **repeat**
4: $R\leftarrow R+1$ or $S\leftarrow S+1$
5: $\mathcal{Z}\leftarrow$**FR-SSPC(**$\mathcal{T},\Omega,p,\rho,\phi$**)**
6: **until** $\|\mathcal{T}_\Omega-\mathcal{Z}_\Omega\|_F^2\le\varepsilon$
7: **output:** $\mathcal{Z}$

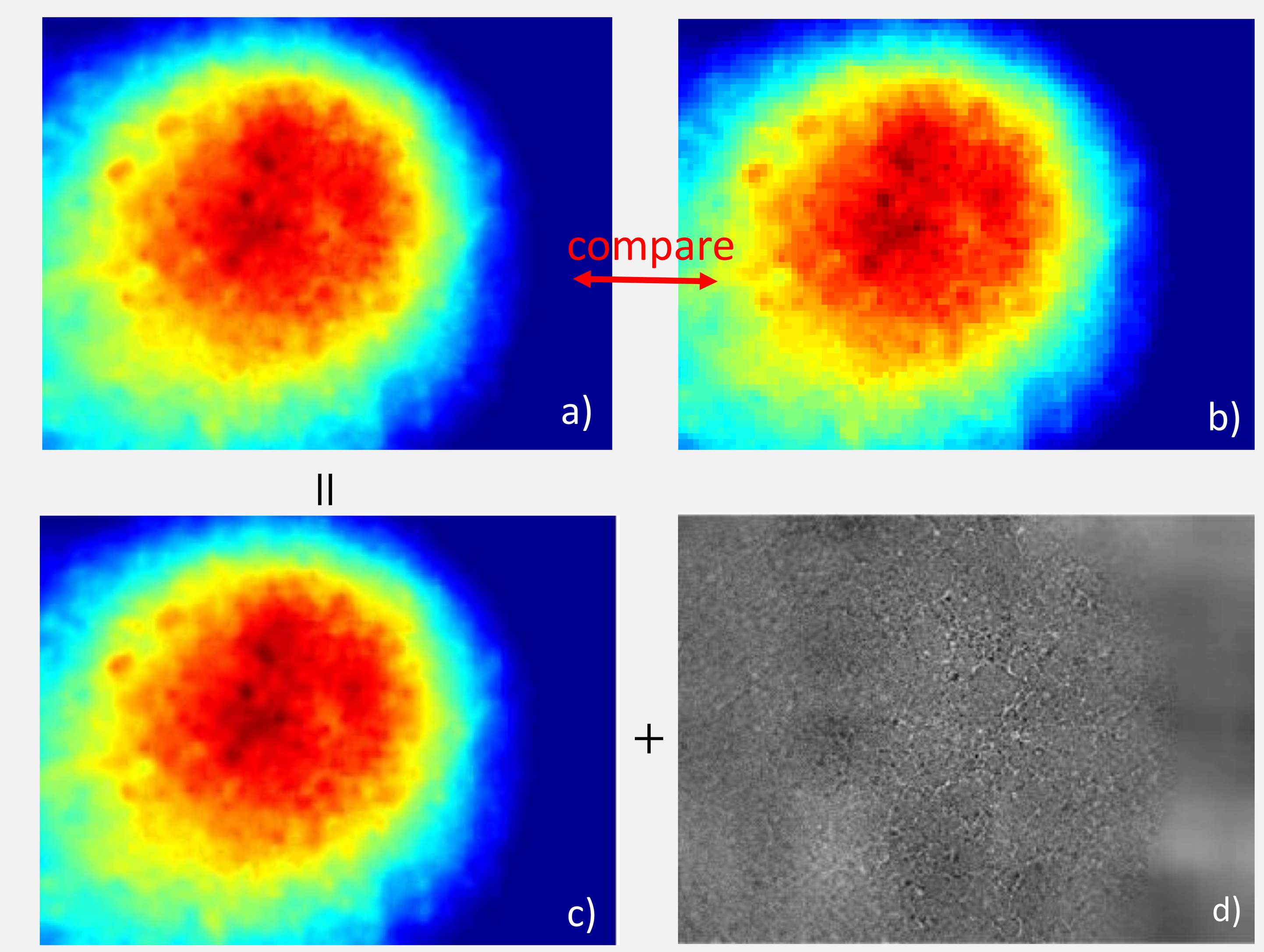


Convergence Behavior of SSPC (Yokota, 2016)

Experimental Results



Convergence performance of SSPC algorithm



a) Reconstructed image; b) Low resolution image; c) $\mathcal{Z}_{smooth}$ image; d) $\mathcal{Z}_{sparse}$ image

Summary and Future Work

- Propose a tensor completion model, which fuses images with different spatial temporal resolution successfully
- Enhance edges in the reconstructed images significantly
- The proposed formulation can be applied to video decoding, inpainting

References

- Yokota, Tatsuya, Qibin Zhao, and Andrzej Cichocki. "Smooth PARAFAC decomposition for tensor completion." *IEEE Transactions on Signal Processing* 64.20 (2016): 5423-5436.
- Liu, Ji, et al. "Tensor completion for estimating missing values in visual data." *IEEE transactions on pattern analysis and machine intelligence* 35.1 (2012): 208-220.

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